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Selection Principles and Pattern Formation in Fluid Mechanics and Nonlinear Shell Theory

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The major research effort of the Principal Investigator during the indicated period has been devoted to a study of "symmetry breaking" in the Taylor problem, and a general study of various spiral-flow problems. The study of spiral flows is part of a general investigation of vortex breakdown.

I. The Taylor Problem. A new approach for determining steady flows of the Taylor problem has been developed to analyze the experimental results of Benjamin [1] and Benjamin & Mullin [2], and a paper [13] has been submitted for publication. The approach makes use of a "structure" parameter and leads to a bifurcation diagram of the type shown in Figure 1 for flows with two and four cells and a bifurcation diagram of the type shown in Figure 2 for flows with one and two cells. One assumes that the fluid fills the space between two concentric cylinders with radii R_1 and R_2, $R_2 > R_1$, and both of length l. The inner and outer cylinders are rotated at constant speeds Ω_1 and Ω_2, respectively. Set

$$\mu = \Omega_2/\Omega_1,\ \ \eta = R_1/R_2,\ \ d = R_2 - R_1,\ \text{ and }\ A = \frac{\Omega_1(\mu-\eta^2)}{1-\eta^2}.$$

The Taylor number, T, and the structure parameter, γ, are defined by the formulas

$$(1) \qquad T = -\frac{1}{2}\,(1+\mu)\left[\frac{4A\,\Omega_1 d^4}{\nu^2}\right], \quad \gamma = \frac{2(\eta^2-\mu)}{\eta(1+\eta)(1+\mu)}.$$

The Taylor number T is a typical "load" parameter for the Taylor problem, however, the use of the structure parameter γ appears to be new. The parameter γ is a measurement of how far away certain physical parameters are from the Rayleigh line (see, e.g., [5, p. 138] for a discussion of the role of the Rayleigh line in the Taylor problem).

The results obtained for steady flows of the Taylor problem by the structure parameter approach, such as those indicated in Figure 1 and Figure 2, are in almost complete agreement with the experimental results in [1; 2], and are more complete than results obtained by means of amplitude equations [3], catastrophe theory [1], or singularity theory [14].

II. <u>Rotating channel flows</u>. The study of rotating channel flows is a preliminary step in the study of spiral flows in the next section. These problems have some important features in common with spiral flows but are somewhat easier to analyze than the spiral flows in Section III.

A. <u>Rotating Couette flow</u>. A paper [7] (with G. Knightly) on combined rotating Couette-Poiseuille channel flow has been completed in which it is shown that the superposition of a Poiseuille flow on a rotating Couette channel flow is destabilizing. In the case of non-rotating Couette-Poiseuille flow, a result of this type was conjectured in [11] on the basis of numerical calculations for the linearized Navier-Stokes equations. The paper [7] develops one possible version of the structure parameter approach for fluid mechanics; a second version of the approach is used in Section IIB to study the opposite problem of the superposition of a Couette flow on a rotating Poiseuille flow.

If U_c (respectively, U_p) denotes the maximum velocity of pure Couette flow (respectively, pure Poiseuille flow) along the axis of the channel when there is no Poiseuille flow present (respectively, no Couette flow present), if Ω denotes the rate of rotation about an axis perpendicular to the plane of mean flow (e.g., see the set up in [4; 8]), and if l denotes the "width" of the channel, then the swirl-like parameter, S_c, and a Reynolds number, R, are defined by

$$(2) \qquad S_c = \frac{l\Omega}{U_c} ,$$

$$(3) \qquad R = \frac{lU_c}{\nu} .$$

For <u>fixed</u> S_c, $0 < S_c < 1/2$, the appropriate structure parameter, γ, and load parameter, λ, are defined by

$$(4) \qquad \gamma = \frac{U_p}{U_c(1-2S_c)} ,$$

$$(5) \qquad \lambda = [2S_c(1-2S_c)]^{1/2}R.$$

The structure parameter γ can now be used as an amplitude parameter to carry out a nonlinear analysis of the problem. As in [6; 13], the scaling for λ is of the form

$$(6) \qquad \lambda = \mu_0 - \mu_0\gamma^2(\mu_0^2 b - \tau), \quad \tau \in I\!R^1 ,$$

where μ_0 is the smallest, positive, characteristic value of the linearized base problem when $\gamma = 0$, b is a known constant, and τ is to be determined. Note that, for fixed U_c and S_c, γ is a measurement of the component of

Poiseuille flow so that the structure parameter approach provides a natural way to analyze the effects of the superposition of a Poiseuille flow on a rotating Couette channel flow. In the next section a completely different choice of structure parameter is made in order to analyze the effects of the superposition of a Couette flow on a rotating Poiseuille channel flow.

B. Rotating Poiseuille flow. A second paper is in preparation on the effects of the superposition of a Couette flow on a rotating Poiseuille channel flow; such an analysis also leads to new results for rotating Poiseuille flow when there is no Couette flow present (e.g., see [4; 8]).

Let U_c, U_p, Ω and l be defined as in Section IIA above. We now define, however, a swirl-like parameter, S_p, and a Reynolds number, R, as

(7) $$S_p = \frac{l\Omega}{U_p},$$

(8) $$R = \frac{lU_p}{\nu}.$$

The appropriate structure parameter, γ, is then defined as

(9) $$\gamma = S_p - \frac{1}{2}\,(1 - \frac{U_c}{U_p}).$$

Note that if $U_c = 0$, i.e., the problem of pure Poiseuille flow, then for γ near $\gamma = 0$ we have S_p near $S_p = \frac{1}{2}$. The values $S_p \simeq \frac{1}{2}$ correspond to those rotation rates in the experiments in [4] for which roll-like instabilities with non-dimensional wave number "approximately" five were observed. Since the numerical calculations in [8] indicate that the minimum Reynolds number for

$S_p = \frac{1}{2}$ in the linearized problem corresponds to a non-dimensional wave number of approximately five, the nonlinear analysis here in the special case when $U_c = 0$ leads to results that are in very close agreement with the experimental results in [4]. The results obtained here for $U_c \neq 0$ are quite different than those obtained in [11; 12] for non-rotating Couette-Poiseuille flow and show the strong effects of rotation.

The analysis in this section is based upon a modified structure parameter approach that is suitable for studying problems in which λ is of the form

(10) $$\lambda = \mu_0 + \mu_0\gamma[\mu_0 a - \tau], \quad \tau \in \mathbb{R}^1 ,$$

rather than the form given in (6). A modified structure parameter approach is required also in the next section to study spiral flows in a cylindrical annulus and, in fact, the modified approach used here was developed originally for the study of such spiral flows.

III. Spiral flows. Various spiral flows have been considered but most of the actual analysis has been carried out for the spiral flows described in [5, Chap. VI]. Such spiral flows have been chosen, in particular, as a natural class of flows for testing Ludwieg's proposal for vortex breakdown.

A. Rotating Couette flow. If we use the set up described in [5, §51ff.], one may formally apply a modified structure parameter approach using a structure parameter, γ, that is proportional to $\sin(\chi - \psi)$, where χ is the spiral angle of the basic flow and ψ is the spiral angle of the disturbed flow. If, e.g., the disturbances are axisymmetric (i.e., $\psi = \pi/2$), then γ is proportional to the axial rate of sliding, U_c. On the other hand, if $\chi = 0$, then one has formally a situation analogous to but more complicated than that in Section IIA. Provided that such a formal approach could be justified, it would show, in particular, that a vanishingly small axial shear would be sufficient to destabilize a pure, stable, swirling flow; this is the same type of result as that obtained in [9] using a formal inviscid analysis. The indicated approach would lead also to spiral flows that are of the same type as those obtained in the experiments in [10]. Such spiral flows seem to be the natural class of flows in which to test Ludwieg's proposal for vortex breakdown because they are probably the simplest class of non-axisymmetric flows that occur in practice.

B. Rotating Poiseuille flow. If we use the general set up in [5, Chap. VI] with $U_c = 0$ and $\Omega_1 = \Omega_2$, one is led formally to a situation somewhat analogous to Section IIB. In the case of spiral flows, however, the appropriate swirl-like parameter no longer varies near $S_p = \frac{1}{2}$ and the analysis appears to be considerably more difficult than that in Section IIB. Nevertheless, the use of Rayleigh's criterion (e.g., see [5]) again suggests an appropriate structure parameter so that one can proceed with a modified structure parameter approach. One would like here, in particular, to obtain spiral flows that are in agreement with the experimental results in [5, §46] when $\Omega_1 = \Omega_2$; if $\Omega_1 \neq \Omega_2$, then the bifurcating spiral vortices are apparently unsteady in every coordinate system and a different type of analysis must be used.

It is expected that the main efforts of the Principal Investigator during the next six months will be devoted to the completion of the work on rotating Poiseuille channel flows outlined in Section IIB, the justification of the formal approaches to spiral flows indicated in Section III, the determination of the exact role of Rayleigh's criterion in spiral flow problems and the investigation of Ludwieg's proposal for vortex breakdown.

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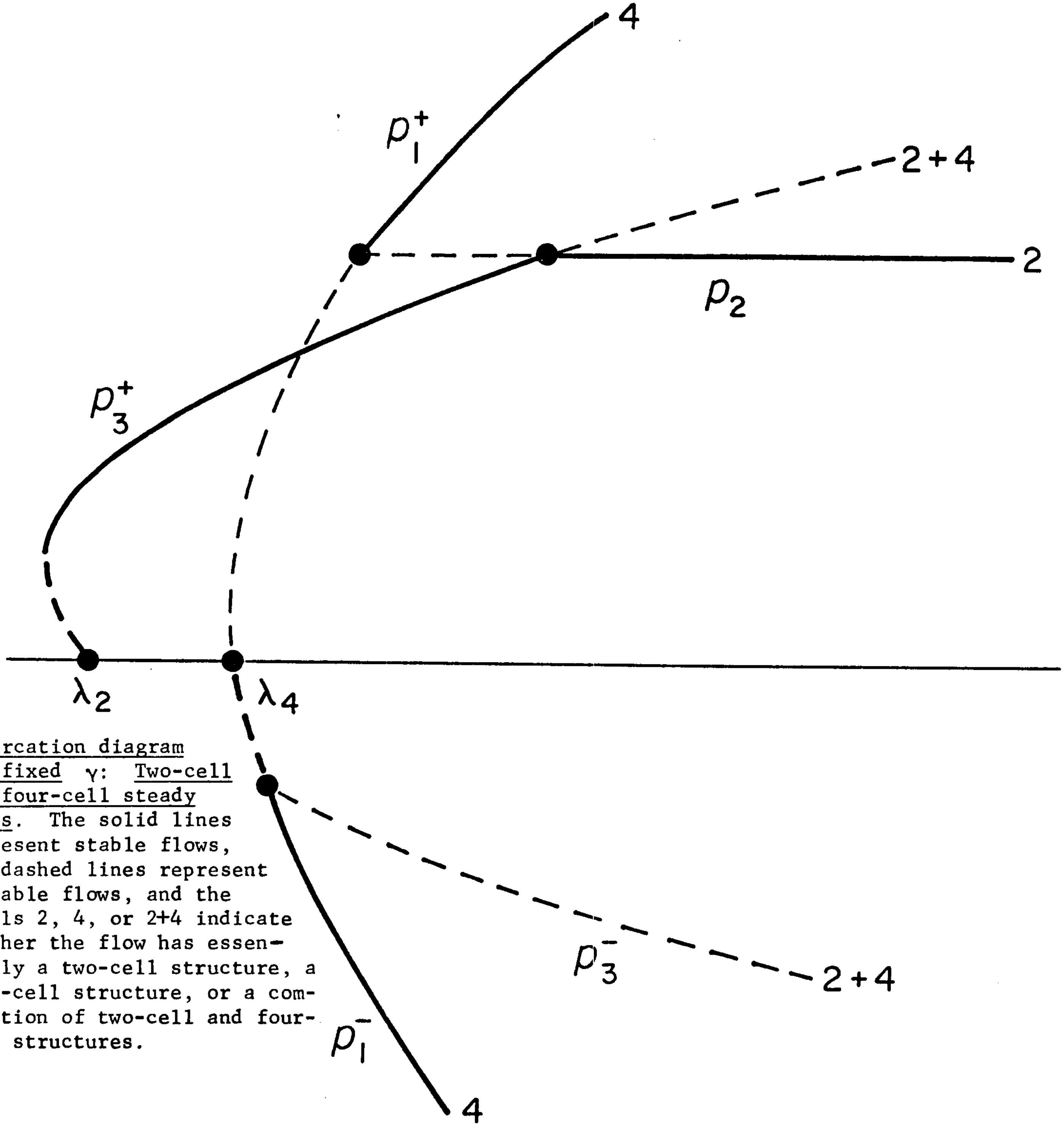

Figure 1. Bifurcation diagram for fixed γ: Two-cell and four-cell steady flows. The solid lines represent stable flows, the dashed lines represent unstable flows, and the labels 2, 4, or 2+4 indicate whether the flow has essentially a two-cell structure, a four-cell structure, or a combination of two-cell and four-cell structures.

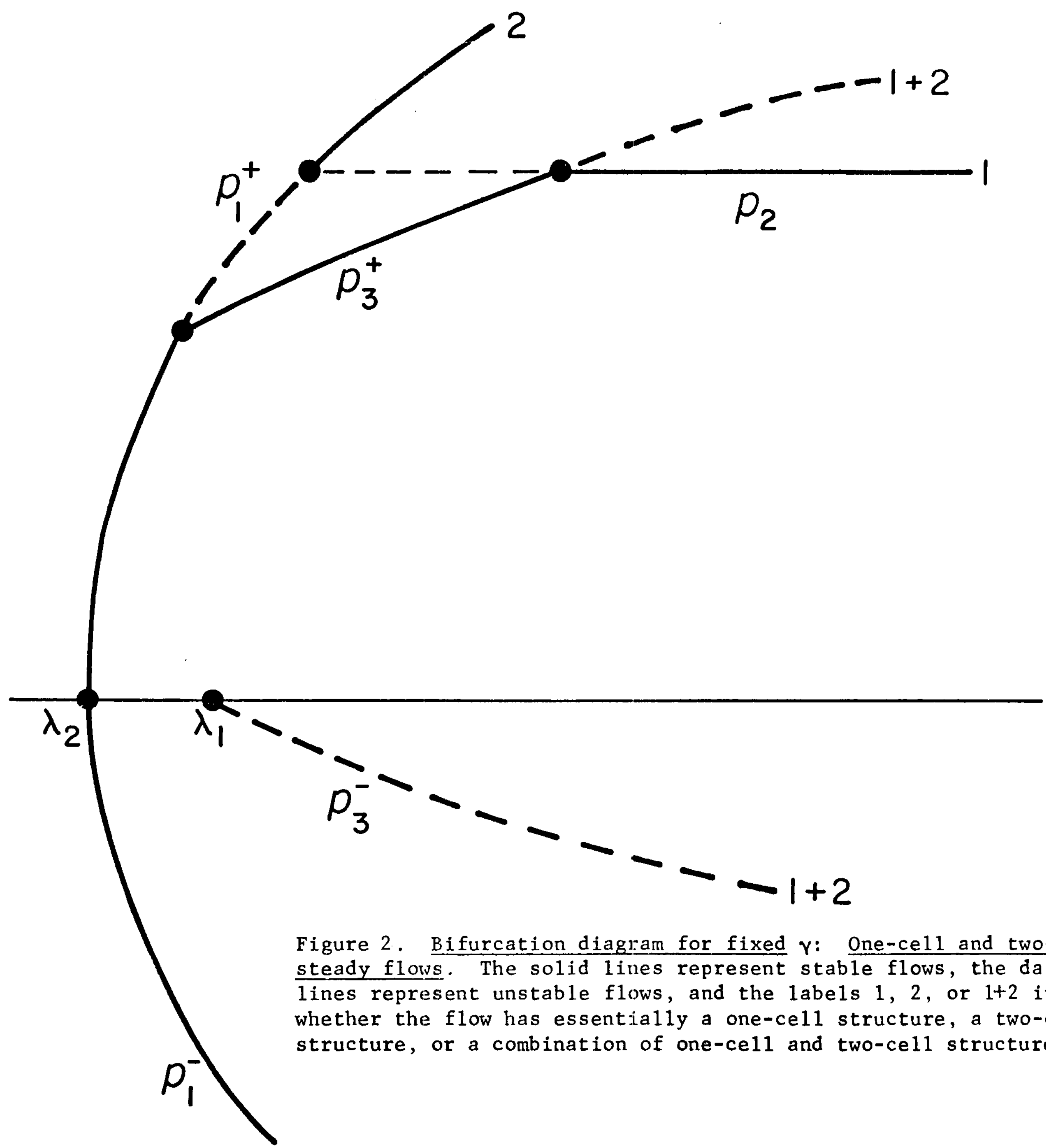

Figure 2. Bifurcation diagram for fixed γ: One-cell and two-cell steady flows. The solid lines represent stable flows, the dashed lines represent unstable flows, and the labels 1, 2, or 1+2 indicate whether the flow has essentially a one-cell structure, a two-cell structure, or a combination of one-cell and two-cell structures.